

Problem #10

For the system described by $\ddot{x} + 25x = 0$:

- a. Determine the characteristic equation.

$$r^2 + 25 = 0$$

- b. Determine the natural frequency, ω_n .

$$\omega_n = \sqrt{25} = 5$$

- c. Determine the damping ratio, ζ .

$$2\zeta\omega_n = 0 \quad \rightarrow \quad \zeta = 0$$

- d. Determine the numerical values of the two roots of the characteristic equation.

$$r^2 + 25 = (r + j5)(r - j5) \quad \rightarrow \quad r = \pm j5$$

- e. Classify the system as *overdamped*, *critically damped*, *underdamped*, or *undamped*.

The system is undamped ($\zeta = 0$)

- f. Assuming $x(0) = 5$ and $\dot{x}(0) = 5$, determine the solution of the given equation.

$$x(t) = K_1 \cos 5t + K_2 \sin 5t$$

$$\dot{x}(t) = -5K_1 \sin 5t + 5K_2 \cos 5t$$

$$\left. \begin{aligned} x(0) = K_1 &= 5 \\ \dot{x}(0) = 5K_2 &= 5 \end{aligned} \right\} \begin{aligned} K_1 &= 5 \\ K_2 &= 1 \end{aligned}$$

$$\therefore x(t) = 5 \cos 5t + \sin 5t, \quad t \geq 0$$